

Microwave Photodetection with Electro-Opto-Mechanical Systems

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While detection of optical photons is today achieved with very high efficiencies, the detection of microwave fields at the photon level still poses non-trivial experimental challenges. In this Letter we propose a model of microwave photodetector which is based on the use of an electro-opto-mechanical system. Using state of the art technology, we show how single microwave photons can efficiently be converted into optical photons which are then measured by standard optics. The overall quantum efficiency of our microwave detector tends to one for achievable values of the optical and microwave cooperativity parameters. Our scheme could be used for sensing and scanning very faint microwave fields, as they may occur in radio astronomy, satellite and deep space communications.

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Introduction. Detection of photons is required for many fundamental and technological purposes. It is particularly relevant in the quantum regime, where the detection of few photons must be resolved, for both photocounting and access to the statistics of radiation field states [1–3]. In the optical, near optical, and infrared domain, photodetection it is readily available through semiconductor technologies, where single photon resolution has been achieved. However microwave frequencies of the radiation are not directly reachable due to technological limitations. Several applications in cavity quantum electrodynamics (QED) have employed Rydberg atoms population detection as an indirect way of determining the coherence of Fabry-Perrot microwave cavity fields, but inevitably the atoms detection induce back-action over the field states [4].

Nowadays a large effort has been devoted to circuit QED, where transmission line resonators are coupled to superconducting devices, known as ‘artificial atoms’. Similarly to the Fabry-Perot cavities, indirect detection of charge or flux populations of the artificial atoms is employed as a way to infer physical properties of the radiation field [5]. Unfortunately, the access to the statistical properties is only available when one can also access the number of microwave photons. Recent proposals for microwave photodetection using circuit QED have been demonstrated with limited quantum efficiency considering nowadays available superconducting technology [6–9].

Moreover it would be interesting to have a photodetector able to work on demand in many different experimental situations and not restricted to a specific setup, similarly to the photodetectors used in the optical domain. One possible strategy to achieve this goal and circumvent both the limited detection efficiency and the problem of detection backaction is to perform a conversion of microwave photons into optical ones, where efficient photon-detection is readily available. This is indeed the strategy suggested in our Letter.

We propose a microwave photodetector which is based on the efficient conversion of microwave photons into op-

tical. This is realized by using an electro-opto-mechanical system where a mechanical oscillator induces strong coupling between a microwave and an optical resonator field. Under specific detuning of both the microwave and optical pumping fields, the average number of optical photons in the signal output is directly proportional to the mean number of microwave photons in the signal input. The proportionality parameter is an effective quantum efficiency which depends on optical and microwave cooperativity parameters associated with the system.

Adopting achievable experimental parameters for the electro-opto-mechanical transducer, we show that the efficiency of the microwave-optical conversion can be extremely high. Combining this feature with the use of optical photodetection on the converted optical photons, we can achieve an overall quantum efficiency which is close to 100%. In our proposal there is no specific requirement for the input microwave signal, so that the setup can be used for microwave photon detection under several circumstances. In particular, it could be used for sensing faint microwave fields at the photon or sub-photon level, with potential applications which include the environmental scanning of electrical circuits, radio astronomy, satellite and deep space communications.

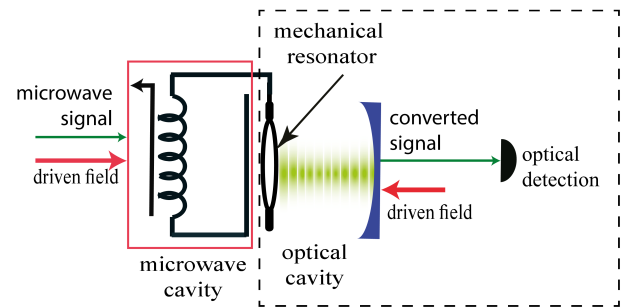


FIG. 1. Schematic description of our microwave photodetector based on an electro-opto-mechanical converter. The microwave signal photons are first converted into optical photons and then measured by a standard optical photodetector.

System. As sketched in Fig. 1, the system consists of two microwave and optical cavity modes, coupled to a single mechanical resonator [11–13], with frequency ω_M and damping rate γ_M . The microwave (optical) cavity works at frequency $\omega_w(\omega_o)$ with the total cavity decay rate being $\kappa_j = \kappa_j^{\text{ext}} + \kappa_j^{\text{int}}$. We include the intrinsic losses with rates κ_j^{int} ($j = w, o$) for both optical and microwave cavity modes, while κ_j^{ext} denote the coupling rates of the respective input ports.

The optical and microwave cavity photons interact with the phononic modes of the mechanical via radiation pressure forces. This interaction is described by the following Hamiltonian [10, 11]

$$\begin{aligned} \hat{H} = & \hbar\omega_M \hat{b}^\dagger \hat{b} + \hbar \sum_{j=w,o} \left[\Delta_j + g_j(\hat{b} + \hat{b}^\dagger) \right] \hat{a}_j^\dagger \hat{a}_j \\ & + i\hbar \sum_{j=w,o} E_j(\hat{a}_j^\dagger - \hat{a}_j), \end{aligned} \quad (1)$$

where \hat{b} is the annihilation operator for mechanical resonator, \hat{a}_j is the annihilation operator for the cavity $j = w, o$, and g_j is the single photon electro-opto-mechanical coupling rate between cavity j and mechanical resonator. Here we also assume that the microwave and optical cavities are driven at the frequencies $\omega_{d,j} = \omega_j - \Delta_j$, where Δ_j is the detuning of the cavity $j = w, o$ and E_j are the amplitudes of the driven pumps.

We can linearize the previous Hamiltonian by expanding the cavity modes around the steady state field amplitudes in each resonator. This is equivalent to set $\hat{c}_j = \hat{a}_j - \sqrt{N_j}$, where $N_j = |E_j|^2/(\kappa_j^2 + \Delta_j^2) \gg 1$ are the strength of the pumps, expressed in terms of mean number of cavity photons induced by the microwave and optical pumps [10, 14]. The effective Hamiltonian of the system, in an interaction picture with respect to the free Hamiltonian, is therefore given by

$$\hat{H} = \hbar G_o(\hat{c}_o \hat{b}^\dagger + \hat{b} \hat{c}_o^\dagger) + \hbar G_w(\hat{c}_w \hat{b}^\dagger + \hat{b} \hat{c}_w^\dagger), \quad (2)$$

where $G_j = g_j \sqrt{N_j}$ are many-photon optomechanical couplings.

In the Hamiltonian of Eq. (2) we have set the cavity detunings to be $\Delta_w = \Delta_o = \omega_M$ and assumed the regime of fast mechanical oscillations, so that we are in the resolved sideband regime for both cavities, with red sideband driving for both microwave and optical cavities. In this regime we have neglected the fast oscillating terms proportional to $\pm 2\omega_M$. The first term in Eq. (2) describes a beam-splitter like interaction between the mechanical resonator and propagating optical fields in the fibre at a rate $F_o := G_o^2/\kappa_o$. Similarly the second term describes the coherent exchange of the excitations between the mechanical resonator and the cavity microwave field at a rate $F_w := G_w^2/\kappa_w$. Note that the process of the exchange of excitations is coherent, as long as $\gamma_M k_B T/(\hbar\omega_M) < F_j$ ($j = w, o$) [14, 15], where k_B is Boltzmann constant, and T is the temperature of the electro-opto-mechanical converter.

By using quantum Langevin equations [17] and standard input-output theory [1], the output variable of the optical cavity $\hat{c}_{o,\text{out}} = \sqrt{2\kappa_o^{\text{ext}}}\hat{c}_o - \hat{c}_{o,\text{ext}}$ is given by

$$\begin{aligned} \hat{c}_{o,\text{out}}(\omega) = & -A(\omega)\hat{c}_{o,\text{ext}} - B(\omega)\hat{c}_{w,\text{ext}} - C(\omega)\hat{b}_{\text{in}} \\ & -D(\omega)\hat{c}_{w,\text{int}} - E(\omega)\hat{c}_{o,\text{int}}, \end{aligned} \quad (3)$$

where the coefficients A, B, C, D and E satisfy $|A|^2 + |B|^2 + |C|^2 + |D|^2 + |E|^2 = 1$ and depend on the cooperativity terms $\Gamma_j = G_j^2/(\kappa_j\gamma_M)$ (see [18] for details).

Pulse conversion. We assume the incoming microwave pulse signal is a coherent pulse with non-zero frequency spread W . We choose such a coherent pulse to have its center frequency at the bare resonance frequency of the microwave cavity, i.e., $\omega_p = \omega_w$. Therefore, the photon flux per unit angular frequency (power spectrum) is given by $\bar{n}_{w,\text{ext}}(\omega) = \langle \hat{c}_{w,\text{ext}}^\dagger(\omega)\hat{c}_{w,\text{ext}}(\omega) \rangle = |\alpha(\omega)|^2$, where

$$\alpha(\omega) = \frac{\alpha_0 e^{-(\omega-\omega_p)^2/W^2}}{(2\pi)^{1/4} \sqrt{W/2}},$$

with α_0 providing the total number of photons in the input pulse via $n_p = \int d\omega \bar{n}_{w,\text{ext}}(\omega) = |\alpha_0|^2$. We see that $\bar{n}_{w,\text{ext}}(\omega)$ is Gaussian [19] with standard deviation $W/2$.

From Eq. (3) we can compute the power spectrum of the field at the output of the optical cavity, i.e., $\bar{n}_{o,\text{out}}(\omega) = \langle \hat{c}_{o,\text{out}}^\dagger(\omega)\hat{c}_{o,\text{out}}(\omega) \rangle$. This is given by

$$\bar{n}_{o,\text{out}}(\omega) = |B(\omega)|^2 \bar{n}_{w,\text{ext}}(\omega) + N_{\text{noise}}(\omega),$$

which depends on the microwave spectrum $\bar{n}_{w,\text{ext}}(\omega)$ and

$$N_{\text{noise}} = \left[|C(\omega)|^2 n_b^T + |D(\omega)|^2 n_w^T + (|A|^2 + |E|^2) n_o^T \right] \delta(\omega)$$

corresponding to Markovian noise added by the conversion process. Here n_b^T and n_w^T represent the thermal numbers of the mechanical and microwave baths, respectively, while $n_o^T = [\exp(\hbar\omega_o/k_B T) - 1]^{-1} \approx 0$ is the thermal occupation number for the intracavity optical field. The mean photon number of the output optical field is equal to $\bar{N}_o = \int d\omega \bar{n}_{o,\text{out}}(\omega)$.

In order to see the effect of the converter on the shape of the incoming pulse, we plot in Fig. 2 both the incoming (microwave) pulse and converted (optical) pulse with respect to the normalized frequency ω/ω_M . In this figure we have taken experimental-achievable parameters for the electro-opto-mechanical converter [14, 16] which is assumed to operate at cryogenic temperatures ($T = 4\text{K}$). We see how a faint microwave pulse ($n_p = 4$ photons) is successfully converted into an output optical pulse.

To analyze the general efficiency of the converter, we consider the ratio between the output optical photons and the input microwave photons, i.e., \bar{N}_o/n_p . This ratio depends on the values of the two cooperativity parameters Γ_w and Γ_o . As we can see from Fig. 3, the larger these parameters are, the better is the microwave to optical conversion, which rapidly approaches the ideal conversion rate of 100%.

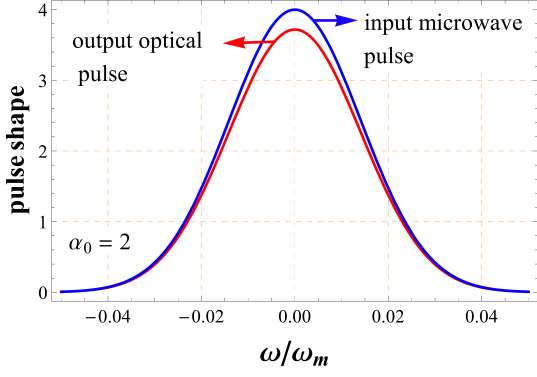


FIG. 2. The power spectrum of the incoming microwave pulse $\bar{n}_{w,\text{ext}}(\omega)$ and that of the converted optical pulse $\bar{n}_{o,\text{out}}(\omega)$ are plotted versus the normalized frequency ω/ω_M . Here, we have assumed a mechanical resonator with frequency $\omega_M/2\pi = 10$ MHz, quality factor $Q = 36 \times 10^4$ and mass $m = 10$ ng, which interacts with a microwave cavity with $\omega_w/2\pi = 10$ GHz, $\kappa_w = 0.101\omega_M$, driven by a microwave source with power $P_w = 35$ mW. We have then considered an optical cavity of length $L = 1$ mm and damping rate $\kappa_c = 0.301\omega_M$, which is driven by a laser with wavelength $\lambda_{0c} = 1064$ nm and power $P_c = 5$ mW. The whole system is located at the cryogenic temperature $T = 4$ K. We have assumed the total number of photons in the incoming microwave pulse is $n_p = 4$ and its bandwidth $W = 1.7$ MHz. This is less than the bandwidth of the electro-opto-mechanical converter, i.e., $W = 0.1W_c$, where $W_c = |\gamma_M(1 + \Gamma_w + \Gamma_o)|$.

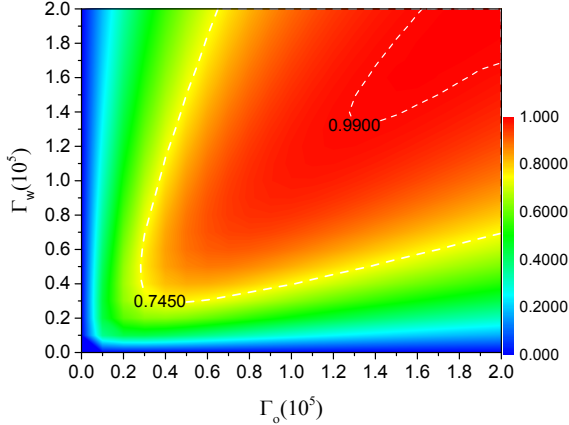


FIG. 3. Ratio between output optical photons and input microwave photons \bar{N}_o/n_p plotted versus the two cooperativity parameters Γ_w and Γ_o . The parameters are the same as Fig. 2.

Quantum efficiency of the microwave detector. At the output of the converter, the optical photons are measured by a photodetector with high efficiency η and large bandwidth (up to 10 GHz, i.e., much larger than W_c). Assuming an incoming microwave pulse with small bandwidth (so that we can approximate the Gaussian pulse with a delta function), the mean number photons which

are detected at the optical output is equal to

$$\bar{N}_o \simeq \eta_{\text{eff}} n_p + \eta N_{\text{thermal}}, \quad (4)$$

where $\eta_{\text{eff}} := \eta |B(0)|^2$ is the quantum efficiency of the microwave detector, and $N_{\text{thermal}} = |C(0)|^2 n_b^T + |D(0)|^2 n_w^T$.

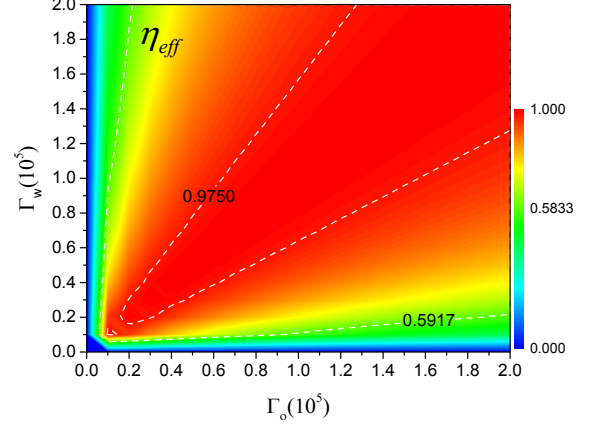


FIG. 4. The effective quantum efficiency of the microwave detector η_{eff} with respect to the cooperativity parameters Γ_w and Γ_o . The other parameters are the same as Fig. 2.

From Eq. (4) we can see that the mean number of detected optical photons has a term $\eta_{\text{eff}} n_p$ which is proportional to the mean number of photons which were present in the input microwave pulse. This is equivalent to having a beam-splitter with transmissivity η_{eff} mixing the incoming microwave field with vacuum fluctuations. Then there is an additional term ηN_{thermal} which accounts for the Markovian thermalization occurring in the conversion process, and depending on the thermal baths of the mechanical oscillator and intracavity microwave field.

Working at cryogenic temperatures (e.g., 4 K), the thermalization term N_{thermal} can be neglected and the effective quantum efficiency of the microwave detector is indeed $\eta_{\text{eff}} = \eta |B(0)|^2$, where $B(0)$ accounts for the strength of the coupling between the optical and microwave fields with the mechanical resonator, which is monotonically increasing in the two cooperativity terms. Assuming a high-efficient optical detector $\eta \simeq 1$ and other achievable experimental conditions, the quantum efficiency η_{eff} can approach 100% for sufficiently high values of the cooperativity parameters, as shown in Fig. 4.

Conclusions. We have designed a model of microwave photodetector whose working mechanism is based on the use of an electro-opto-mechanical converter. In our approach, microwave fields with a small number of photons can efficiently be converted into optical fields, which are then subject to standard optical measurements. We have shown that overall quantum efficiency of our detector can be very high assuming cryogenic temperatures and achievable experimental parameters. Our receiver can potentially be used in all those scenarios connected with

the detection of faint microwave signals, including deep space communications and radio astronomy, for instance, in the mapping of the cosmic background radiation.

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Note added. Upon completion of our manuscript, we noted that a similar work by Zhang *et al.* has been recently submitted to the arxiv (arXiv:1410.0070).

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- [17] The dynamics of the system can be described by following quantum Langevin equations

$$\dot{\hat{c}}_w = -\kappa_w \hat{c}_w - iG_w \hat{b} + \sqrt{2\kappa_w^{\text{ext}}} \hat{c}_{w,\text{ext}} + \sqrt{2\kappa_w^{\text{int}}} \hat{c}_{w,\text{int}},$$

$$\dot{\hat{c}}_o = -\kappa_o \hat{c}_o - iG_o \hat{b} + \sqrt{2\kappa_o^{\text{ext}}} \hat{c}_{o,\text{ext}} + \sqrt{2\kappa_o^{\text{int}}} \hat{c}_{o,\text{int}}$$

$$\dot{\hat{b}} = -\gamma_M \hat{b} - iG_o \hat{c}_o - iG_w \hat{c}_w + \sqrt{2\gamma_M} \hat{b}_{\text{in}},$$

- where \hat{b}_{in} and $\hat{c}_{j,\text{int}}$ are the quantum noise operators of the mechanical resonator and cavity j , respectively, while $\hat{c}_{j,\text{ext}}$ describes the signal fluctuations at the input of cavity j . The noise operators satisfy white-noise correlation functions $\langle \hat{b}_{\text{in}}(t) \hat{b}_{\text{in}}^\dagger(t') \rangle = \langle \hat{b}_{\text{in}}^\dagger(t) \hat{b}_{\text{in}}(t') \rangle + \delta(t - t') = (\bar{n}_b^T + 1) \delta(t - t')$ and $\langle c_{j,\text{int}}(t) c_{j,\text{int}}^\dagger(t') \rangle = \langle c_{j,\text{int}}^\dagger(t) c_{j,\text{int}}(t') \rangle + \delta(t - t') = (\bar{n}_j^T + 1) \delta(t - t')$.

- [18] The explicit expressions of the coefficients are

$$A(\omega) = \frac{[\tilde{\omega}_o - 2\kappa_o^{\text{ext}}/\kappa_o][\Gamma_w + \tilde{\omega}_w \tilde{\omega}_b] + \Gamma_o \tilde{\omega}_w}{\tilde{\omega}_w [\tilde{\omega}_o \tilde{\omega}_b + \Gamma_o] + \Gamma_w \tilde{\omega}_o},$$

$$B(\omega) = 2\sqrt{\frac{\kappa_o^{\text{ext}}}{\kappa_o}} \sqrt{\frac{\kappa_w^{\text{ext}}}{\kappa_w}} \frac{\sqrt{\Gamma_w \Gamma_o}}{\tilde{\omega}_w [\tilde{\omega}_o \tilde{\omega}_b + \Gamma_o] + \Gamma_w \tilde{\omega}_o},$$

$$C(\omega) = \sqrt{\frac{\kappa_o^{\text{ext}}}{\kappa_o}} \frac{2i\sqrt{\Gamma_o} \tilde{\omega}_w}{\tilde{\omega}_w [\tilde{\omega}_o \tilde{\omega}_b + \Gamma_o] + \Gamma_w \tilde{\omega}_o},$$

$$D(\omega) = 2\sqrt{\frac{\kappa_o^{\text{ext}}}{\kappa_o}} \sqrt{\frac{\kappa_w^{\text{int}}}{\kappa_w}} \frac{\sqrt{\Gamma_o \Gamma_w}}{\tilde{\omega}_w [\tilde{\omega}_o \tilde{\omega}_b + \Gamma_o] + \Gamma_w \tilde{\omega}_o},$$

$$E(\omega) = 2\sqrt{\frac{\kappa_o^{\text{int}}}{\kappa_o}} \sqrt{\frac{\kappa_o^{\text{ext}}}{\kappa_o}} \frac{\Gamma_w + \tilde{\omega}_w \tilde{\omega}_b}{\tilde{\omega}_w [\tilde{\omega}_o \tilde{\omega}_b + \Gamma_o] + \Gamma_w \tilde{\omega}_o},$$

where $\tilde{\omega}_j = 1 - i\frac{\omega}{\kappa_j}$ ($j = w, o$) and $\tilde{\omega}_b = 1 - i\frac{\omega}{\gamma_M}$.

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